International Journal of Theoretical Physics, Vol. 36, No. 11, 1997

Signatures in the Angular Power Spectrum of the Cosmic Microwave Background Anisotropies: Topological Defect Scenarios Versus Inflationary Models

Mairi Sakellariadou[1]

Received June 25, 1997

Acoustic peaks in the angular power spectrum of the cosmic microwave background anisotropies may allow us to distinguish among the two classes of models—topological defect scenarios and inflationary models—which attempt to explain the origin of structure formation in the universe. I briefly sketch the main differences between these two classes of models and illustrate the relevant analysis of induced density perturbations, in a model where density perturbations are generated by global scalar fields, within a universe dominated by cold dark matter.

1. INTRODUCTION

A very important question in modern cosmology is the origin of the observed large-scale structure. It is quite widely accepted that it was produced by gravitational instability from small primordial fluctuations in the energy density which were generated in the early universe. Within this framework there are two currently investigated classes of models which address the origin of primordial fluctuations. Initial density perturbations can be due to *freezing in* of quantum fluctuations of a scalar field during a period of inflation (Steinhard, 1995) or they may be seeded by topological defects formed during a symmetry-breaking phase transition in the early universe (Kibble, 1980).

Inflationary fluctuations were created very early in the universe, and were driven far outside the Hubble radius by the dramatic expansion. These fluctuations evolve according to homogeneous linear perturbation equations until rather late times, when, during gravitational collapse, nonlinear effects

[1] Département de Physique Théorique, Université de Genève, CH-1211 Geneva 4, Switzerland.

0020-7748/97/1100-2503$12.50/0 © 1997 Plenum Publishing Corporation

become important. Another characteristic of such perturbations is the way *randomness* comes into the model. Time evolution of inflationary perturbations is linear and deterministic; randomness enters the whole picture only when initial conditions are set up. For these reasons inflationary perturbations are often called *passive* and *coherent* (Magueijo *et al.*, 1996). They lead to an approximately *scale-invariant* (Harrison–Zel'dovich) (Harrison, 1970; Zel'dovich, 1972) spectrum of density perturbations, with a Gaussian distribution of amplitudes on scales which are cosmological today.

For topological defect models, or in general models where perturbations are originated by any inhomogeneously distributed form of energy, which contributes only a small fraction to the total energy density of the universe and interacts with the cosmic fluid only gravitationally, called *seeds*, the picture is quite different and certainly the analysis is more complicated. Seed fluctuations are continuously generated by the defect evolution, which is in general a nonlinear process. Moreover, randomness plays a role not only when initial conditions are set up, but also during the subsequent time evolution. Causality imposes stringest constraints on these models and requires the existence of a large-scale radiation white-noise spectrum. The randomness of the nonlinear evolution of the defect network may destroy the coherence of the seeded fluctuations in the cosmic fluid. Thus, seed perturbations are often referred to as *active* and *incoherent* (Magueijo *et al.*, 1996). They evolve according to linear inhomogeneous perturbation equations, and therefore their study is more involved than the analysis of perturbations within inflationary models. Perturbations from defect models lead also to an approximately scale-invariant spectrum of density perturbations on large angular scales; however, the spectrum is in general non-Gaussian. Topological defect models have the advantage of depending on very few parameters and therefore these models are quite appealing.

Either of these two classes of models predicts precise fingerprints in the cosmic microwave background (CMB) anisotropies, which can be used to differentiate among them using a purely linear analysis.

In this contribution I will briefly sketch the main differences in the angular power spectrum of CMB anisotropies induced by seeds, as compared to those triggered by generic inflationary models, and illustrate a model where density perturbations are sourced by global topological defects in a spatially flat universe dominated by cold dark matter. I am only interested in scalar perturbations.

2. COSMIC MICROWAVE BACKGROUND ANISOTROPIES

The CMB fluctuation spectrum is usually parametrized in terms of multipole moments C_ℓ, defined as the coefficients in the expansion of the temperature autocorrelation function

$$\left\langle \frac{\delta T}{T}(\mathbf{n}) \frac{\delta T}{T}(\mathbf{n}') \right\rangle \Bigg|_{(\mathbf{n}\cdot\mathbf{n}'=\cos\vartheta)} = \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_\ell P_\ell(\cos\vartheta) \tag{1}$$

which compares points in the sky separated by an angle ϑ. I will now describe the main physical mechanisms which contribute to the red-shift of photons propagating in a perturbed Friedmann geometry.

(i) On large angular scales, the main contribution to CMB anisotropies comes from inhomogeneities in the spacetime geometry. These inhomogeneities determine the change in the photon energy due to the difference of the gravitational potential at the position of emitter and observer, and account for red-shifting or blue-shifting caused by the time dependence of the gravitational field along the path of the photon. They are known as *ordinary Sachs–Wolfe* and *integrated Sachs–Wolfe* (ISW) effects, respectively.

(ii) On angular scales $0.1° \lesssim \theta \lesssim 2°$, the main contribution comes from the intrinsic inhomogeneities on the surface of the last scattering due to acoustic oscillations in the coupled baryon–radiation fluid prior to decoupling. On the same angular scales as this acoustic term, there is a Doppler contribution to the CMB anisotropies due to the relative motions of emitter and observer. The sum of these two contributions is denoted by the term *acoustic peaks*.

(iii) On scales smaller than about $0.1°$, the anisotropies are damped due to the finite thickness of the recombination shell, as well as by photon diffusion during recombination (*Silk damping*).

In the case of inflationary models, there has been a large number of studies and a lot of excitement, in particular since CMB anisotropies might lead to a determination of fundamental cosmological parameters, such as the spatial curvature of the universe Ω_0, the baryon density Ω_b, the Hubble constant H_0, and the cosmological constant Λ. At multipoles $\ell \gtrsim 200$, the CMB anisotropies become sensitive to fluctuations inside the Hubble horizon at recombination. Since these fluctuations had enough time to evolve prior to last scattering, they are sensitive to evolutionary effects that depend on a number of cosmological parameters (Hu and Sugiyama, 1995a, b).

Both generic inflationary models and topological defect scenarios predict an approximately scale-invariant (Harrison–Zel'dovich) spectrum of density perturbations on large angular scales. Thus, CMB anisotropies on intermediate and small angular scales are very important. If the two families of models predict different characteristics for the acoustic peaks (e.g., position and amplitude of primary peak, separation between first and second peak, existence or absence of secondary peaks), one can discriminate among them. Inflationary perturbations predict coherent oscillations, with the first acoustic peak at $\ell \sim 200$, having an amplitude $\sim$(4–6) times the Sachs–Wolfe plateau, and the appearance of secondary oscillations (Steinhard, 1995).

Topological defect models, on the other hand, predict in general a quite different power spectrum than inflationary models, due to the behavior of perturbations on superhorizon scales. Causality and scale invariance have quite different implementations in this class of models. In addition, due to the nonlinear defect evolution, studies of CMB anisotropies induced by seeds are quite more complicated than those in inflationary models, and therefore there are just a few studies done, while the predictions of such investigations are quite less precise.

Studies on topological defect models (Magueijo *et al.*, 1996; Crittenden and Turok, 1995; Durrer *et al.*, 1996) show that the primary acoustic peak is located to the right of the adiabatic position at which the peak arises in a generic inflationary model. The value of this shift to smaller angular scales is determined by the coherence length of the defect. Also, the structure of secondary peaks may be quite different for defects as compared to inflation. Depending on whether the defect is effectively coherent or not, which is a direct implication of the constraints imposed by causality on defect formation and evolution, secondary peaks will or will not appear in the power spectrum (Magueijo *et al.*, 1996).

3. ACOUSTIC PEAKS IN A MODEL WITH TEXTURES AND CDM

Employing a gauge-invariant linear perturbation analysis (Durrer, 1994), we calculate (Durrer *et al.*, 1996) the position and relative amplitude of the first acoustic peak in the angular power spectrum of CMB anisotropies produced by global textures, π_3 defects, in a universe dominated by cold dark matter (CDM). Neglecting the integrated Sachs–Wolfe (ISW) effect, the Silk damping, and the contribution of neutrino fluctuations, the acoustic contribution to the CMB anisotropies is

$$\left[\frac{\delta T}{T}(\mathbf{x}, \mathbf{n})\right]^{\text{acoustic}} \approx \frac{1}{4} D_g^{(r)}(\mathbf{x}_{\text{rec}}, t_{\text{rec}}) + \mathbf{V}(\mathbf{x}_{\text{rec}}, t_{\text{rec}}) \cdot \mathbf{n} \tag{2}$$

where $D_g^{(r)}$ is a gauge-invariant variable describing the density fluctuation in the coupled baryon–radiation fluid; $\mathbf{V}$ is the peculiar velocity of the baryon fluid with respect to the overall Friedmann expansion; $\mathbf{x}_{\text{rec}} = \mathbf{x} - \mathbf{n}t_0$, where $\mathbf{n}$ denotes a direction in the sky; and t_0 and t_{rec} are the present (conformal) time and the (conformal) time of recombination, respectively.

We study (Durrer *et al.*, 1996) a two-component fluid system, baryons plus radiation, which prior to recombination are tightly coupled, and CDM.

The evolution for the perturbation variables D (density perturbation) and V (velocity perturbation) in a flat background background is given by

$$\mathfrak{D}\begin{pmatrix} D_g^{(r)} \\ D_g^{(c)} \end{pmatrix} = S \tag{3}$$

where superscripts (r) and (c) indicate the coupled baryon–radiation plasma and CDM, respectively. In the above equation, $\mathfrak{D}$ stands for a second-order differential operator and S denotes the source term, in general given by $S = 4\pi G a^2(\rho + 3p)^{\text{seed}}$, where a stands for the scale factor. In our case, where the seed is described by a global scalar field ϕ, the source term is $S = 8\pi G(\dot{\phi})^2$; an overdot stands for derivative with respect to conformal time t. Numerical simulations show that the average of $|\dot{\phi}|^2$ over a shell of radius k can be modeled by (Durrer and Zhou, 1996)

$$\langle|\dot{\phi}|^2\rangle(k, t) = \frac{(1/2)A\eta^2}{\sqrt{t}[1 + \alpha(kt) + \beta(kt)^2]} \tag{4}$$

where η is the symmetry-breaking scale of the phase transition leading to texture formation; A, α, β are parameters of order 1, specified by numerical simulations. For a given scale k, we chose the initial time such that the perturbation is superhorizon and the universe is radiation-dominated. With these initial conditions we solve the system of second-order equations for the perturbation variables, obtaining $D_g^{(r)}$ and $D_g^{(r)}$. The acoustic contribution to the CMB anisotropies is given by

$$C_\ell = \frac{2}{\pi}\int dk\left[\frac{k^2}{16}|D_g^{(r)}(k, t_{\text{rec}})|^2 j_\ell^2(kt_0) + \frac{1}{(1 + w)^2}|\dot{D}_g^{(r)}(k, t_{\text{rec}})|^2(j_\ell^1(kt_0))^2\right] \tag{5}$$

where an overdot stands for derivative with respect to conformal time t, $w = p_r/\rho_r$, j_ℓ is the spherical Bessel function of order ℓ, and j_ℓ^1 its first derivative. The angular power spectrum yields the acoustic peaks. The ISW effect will shift the position of the first peak to somewhat larger scales, lowering ℓ_{peak} by 5–10% and possibly increasing slightly its amplitude (by less than 30%). So the primary peak is displaced by $\Delta\ell \sim 150$ (Durrer *et al.*, 1996) toward smaller angular scales than in standard inflationary models. Silk damping will decrease the relative amplitude of the third peak with respect to the second one; however, it will not affect substantially the height of the first peak, which is $\ell(\ell + 1)\, C_\ell|_{\text{peak}} = (2\text{–}3) \cdot 6C_2$ (Durrer *et al.*, 1996).

In this analysis (Durrer *et al.*, 1996), as well as in Critenden and Turok (1995), we assumed maximum coherence for the texture models and found that the peaks were preserved. As emphasized in Albrecht *et al.*, (1996), the distinctive appearance of acoustic peaks and troughs seen in inflationary calculations and texture models depend sensitively on the temporal coherence of the sources. Assuming little coherence, the peaks are washed out, while an assumption of total coherence preserves them. An incoherent defect perturbation is effectively coherent and displays secondary oscillations if the defect scaling coherence time is much bigger than $2\pi t \xi_c^{-1}$, where ξ_c is the defect coherence length (Magueijo *et al.*, 1996). Checking numerically whether the unequal time correlator for $|\dot{\phi}|^2$ has an exponential decay on a time scale which will define the coherence time, we conclude that, for textures, unequal time coherence is reasonably well maintained on superhorizon scales.

We now question the validity of the coherence assumption for local gauge strings. Albrecht *et al.*, (1996) assumed that strings were effectively incoherent and obtained a rather featureless CMB power spectrum at large multipoles ℓ. In Magueijo *et al.*, (1996) this assumption was justified by a numerical study of the two-time energy density correlator. The authors concluded the absence of secondary oscillations and the validity of the totally incoherent approximation. Performing numerical experiments, we investigate scaling properties of the power spectra and correlations of the energy and momentum in an evolving string network in Minkowski space (Vincent *et al.*, 1996) and measure the coherence time in the network. We find that the coherence time is smaller than, but of the same order of magnitude as, the period of acoustic oscillations in the photon baryon fluid at decoupling. This is in turn smaller than the time at which the power in the energy and velocity sources peak. We believe that our string correlation functions (Vincent *et al.*, 1996) can serve as realistic sources to answer the question of existence or absence of secondary peaks in the CMB angular power spectrum.

4. CMB ANISOTROPIES INDUCED BY SCALING SEEDS

Within the context of CMB anisotropies induced by seeds, we would like to determine the robust features of the predicted angular power spectrum which are independent of the particular model or the parameters obtained from numerical simulations. Moreover, a crucial question is whether there are models leading to a power spectrum with the same characteristics as those dictated by a generic inflationary model. For example, Turok (1996) showed that there are defect models leading to a primary acoustic peak located at the adiabatic position.

In Durrer and Sakellariadou (1997) we estimate the Sachs–Wolfe and acoustic contributions for models with perturbations triggered by scaling sources, and in particular global scalar fields, in a universe dominated by CDM. Neglecting Silk damping, gauge-invariant linear perturbation analysis leads to

$$\frac{\delta T}{T}(\mathbf{x}, \mathbf{n}) = \left[-\frac{1}{4} D_g^{(r)}(\mathbf{x}) - V_j(\mathbf{x})n^j - (\Psi - \Phi)(\mathbf{x})\right]_i^f + \int_i^f (\dot{\Psi} - \dot{\Phi})(\mathbf{x}', t') \, d\tau \tag{6}$$

where τ denotes physical time; Φ and Ψ are the Bardeen potentials, quantities describing the perturbations in the geometry. The geometrical perturbations Ψ and Φ can be separated into a part coming from standard matter and radiation (denoted by subscript m), and a part due to the seeds (denoted by subscript s):

$$\Psi = \Psi_m + \Psi_s \tag{7}$$

$$\Phi = \Phi_m + \Phi_s \tag{8}$$

where Ψ_s Φ_s are determined by the energy-momentum tensor of the seeds, and Ψ_m, Φ_m satisfy Einstein's equations.

The C_ℓ are found to be

$$C_\ell = \frac{2}{\pi} \int \frac{\langle |\Delta_\ell(\mathbf{k})|^2 \rangle}{(2\ell + 1)^2} k^2 \, dk \tag{9}$$

with

$$\frac{\Delta_\ell}{2\ell + 1} = \frac{1}{4} D_g^{(r)}(\mathbf{k}, t_{rec}) j_\ell(kt_0) - j_\ell^l(kt_0)\mathbf{V}_r(\mathbf{k}, t_{rec}) + k \int_{t_{rec}}^{t_0} (\Psi - \Phi)(\mathbf{k}, t') j_\ell^l(k(t_0 - t')) \, dt' \tag{10}$$

where j_ℓ is the spherical Bessel function of order ℓ, and j_ℓ its first derivative.

Considering that the matter content of the universe is a baryon–radiation fluid, tightly coupled until decoupling, and CDM, the evolution of the perturbation variables until decoupling is described by a system of linear nonhomogeneous equations for the perturbation variables (Durrer and Sakellariadou, 1997)

$$\mathcal{D} \begin{pmatrix} D_g^{(r)} \\ D_g^{(c)} \\ V_r \\ V_c \\ \Psi_m \\ \Phi_m \\ \Psi_s \\ \Phi_s \end{pmatrix} = 0 \tag{11}$$

(the seeds enter the system through the Ψ_s, Ψ_s), where $\mathcal{D}$ is a first-order differential operator. Note that subscripts and or superscripts r, (r) and c, (c) indicate the coupled baryon–radiation plasma and CDM, respectively. To solve the above system of equations, we choose initial conditions such that for a given scale k, the initial time t_{in} is early enough that the perturbations are superhorizon and the universe is radiation-dominated. Analyzing the obtained system of equations for the initial conditions, we find (Durrer and Sakellariadou, 1997) that, since seeds are uncorrelated on superhorizon scales, they are always compensated by the presence of matter and radiation, resulting in nearly isocurvature fluctuations (Ψ, $\Phi << \Psi_s$, Φ_s).

Restricting our study to scaling seeds, for example, global topological defects, we can solve analytically (Durrer and Sakellariadou, 1997) the system of equations for the initial conditions and then integrate numerically (11) for the perturbation variables. We find (Durrer and Sakellariadou, 1997) that the characteristics of the power spectrum depend crucially on the parameters of the model, usually obtained from involved numerical simulations. We obtain power spectra with the primary peak in the adiabatic position and power spectra with the peak displaced in smaller angular scales. Also, the relative height of the primary peak fluctuates according to the particular set of parameters which characterize the behavior of the seed functions, which are gauge-invariant perturbation variables that parametrize the energy density, pressure, scalar velocity potential, and anisotropic stress potential of the seeds. Here we understand as *scaling seeds* those for which the behavior of seed functions, in the absence of any intrinsic length scale other than the cosmic horizon, is determined by dimensional reasons.

5. CONCLUSIONS

Based on observations, we believe that the observed large-scale structure was produced by gravitational instability from small primordial fluctuations in the energy density which were generated in the early universe. Within this framework, the two families of models to explain the origin of primordial density perturbations are inflationary models and topological defect scenarios. Either of these two families of models predicts precise fingerprints in the cosmic microwave background anisotropies, which can be used to differentiate among them using a purely linear analysis. Both families lead to an approximately scale-invariant (Harrison–Zel'dovich) spectrum of density fluctuations on large angular scales. However, the power spectrum predicted from topological defect models seems to have, on smaller angular scales, different characteristics than the ones predicted by a generic inflationary model. In particular, the features of the acoustic peaks predicted by defect models depend crucially on parameters of the considered model. The next satellite experiments, as well as ground-based and balloon experiments, will provide a detailed power spectrum against which we will be able to test the power spectra predicted by our theoretical models. We also expect to be able to determine a number of fundamental cosmological parameters up to a high degree of accuracy.

REFERENCES

Albrecht, A., Coulson, D., Ferreira, P., and Magueijo, J. (1996). *Physical Review Letters*, **76**, 1413.

Crittenden, R. G., and Turok, N. (1995). *Physical Review Letters*, **75**, 2642.

Durrer, R. (1994). *Foundations of Cosmic Physics*, **15**, 209.

Durrer, R., and Sakellariadou, M. (1997). Microwave background anisotropies from scaling seed perturbations, astro-ph/9702028, *Physical Review D* (to appear).

Durrer, R., and Zhou, Z. H. (1996). *Physical Review D*, **53**, 5384.

Durrer, R., Gangui, A., and Sakellariadou, M. (1996). *Physical Review Letters*, **76**, 579.

Harrison, H. R. (1970). *Physical Review D*, **1**, 2726.

Hu, W., and Sugiyama, N. (1995a). *Astrophysical Journal*, **444**, 489.

Hu, W., and Sugiyama, N. (1995b). *Physical Review D*, **51**, 2599.

Kibble, T. W. B. (1980). *Physics Reports*, **67**, 183.

Magueijo, J., Albrecht, A., Ferreira, P., and Coulson, D. (1996). The structure of Doppler peaks induced by active perturbations [astro-ph/9605047].

Steinhard, P. J. (1995). Cosmology at the crossroads, in *Particle and Nuclear Astrophysics and Cosmology in the Next Millenium*, E. W. Kolb and R. Peccei, eds., World Scientific, Singapore.

Turok, N. (1996). *Physical Review D*, **54**, 3686.

Vincent, G., Hindmarsh, M., and Sakellariadou, M. (1997). Correlations in cosmic string networks. *Physical Review D*, **55**, 573.

Zel'dovich, Y. B. (1972). *Monthly Notices of the Royal Astronomical Society*, **160**, 1.